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ROS坐标系和坐标变换

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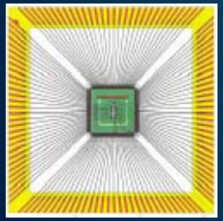
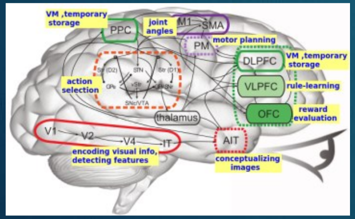
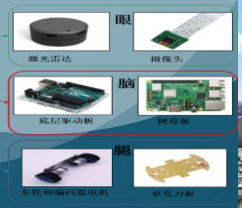
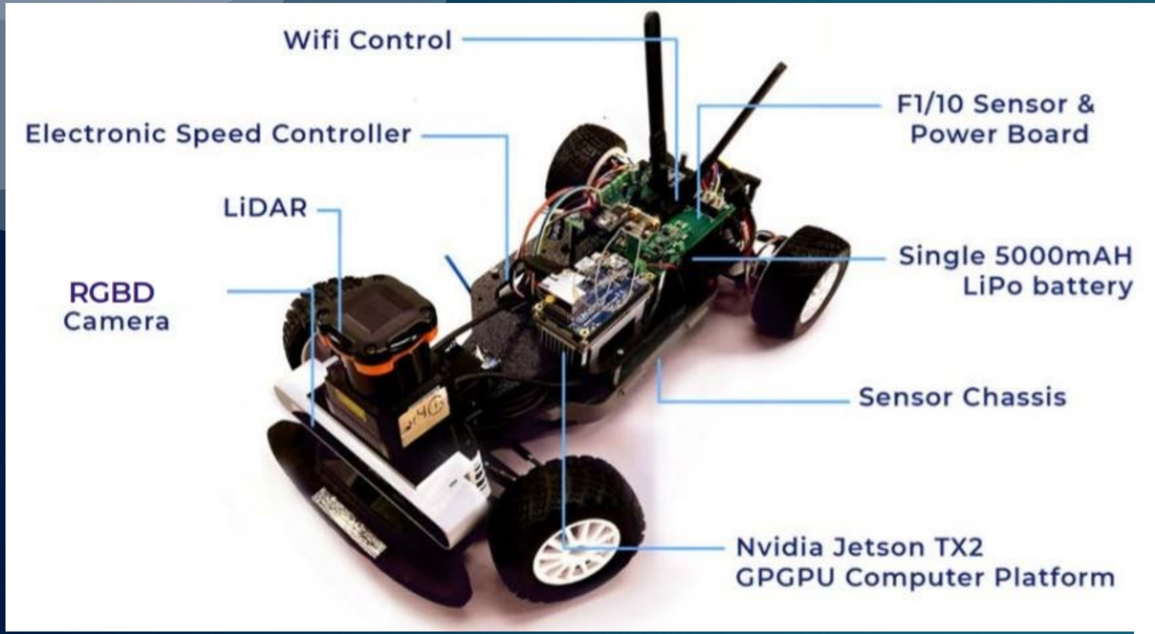






为何我们在机器人上需要坐标系和坐标变换？

传感器在它自身的参考坐标系内给出测量值输出！



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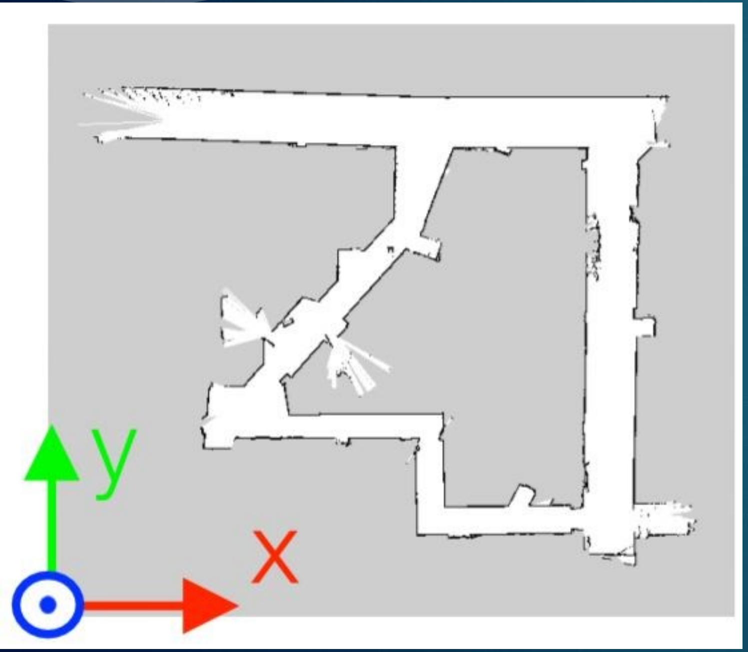
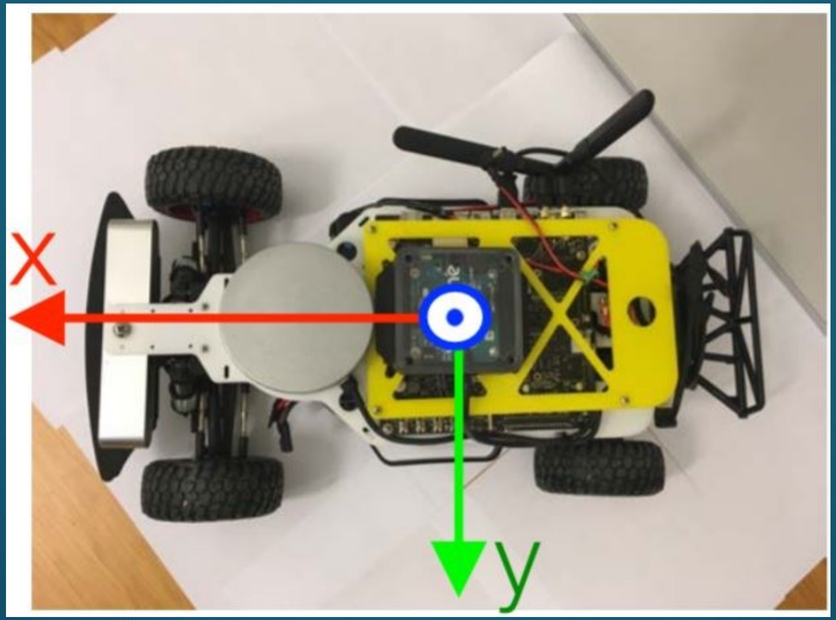
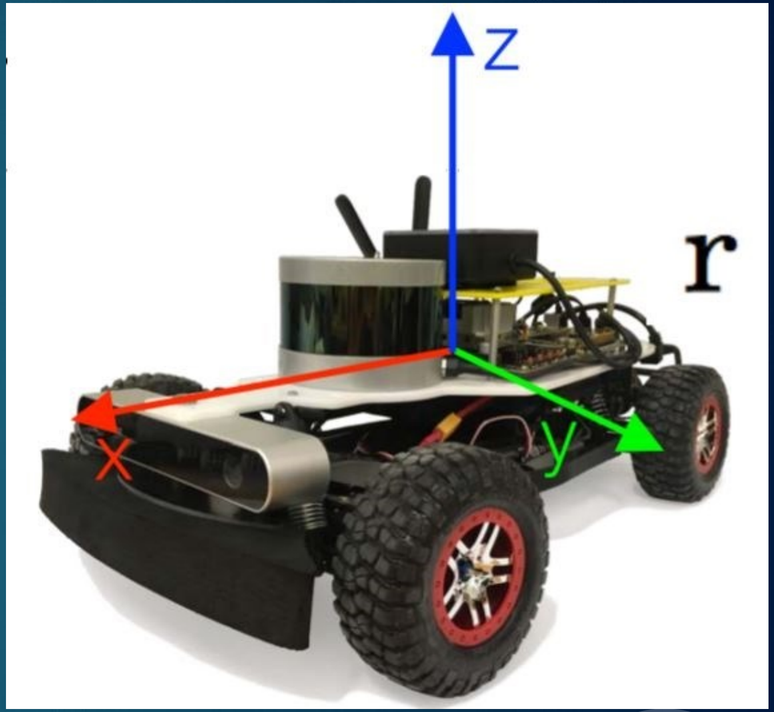
何为坐标系

- A set of orthogonal axes attached to a body that serves to describe position of points relative to that body
- Axes intersect at a point known as the **origin**



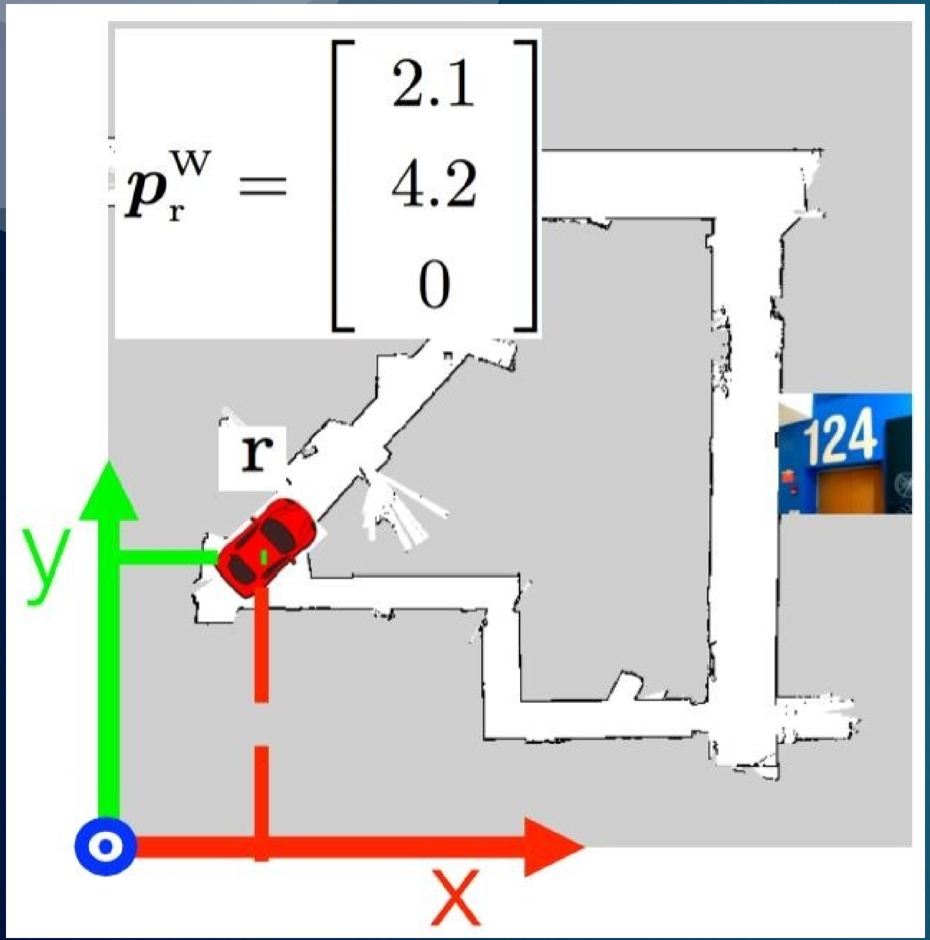


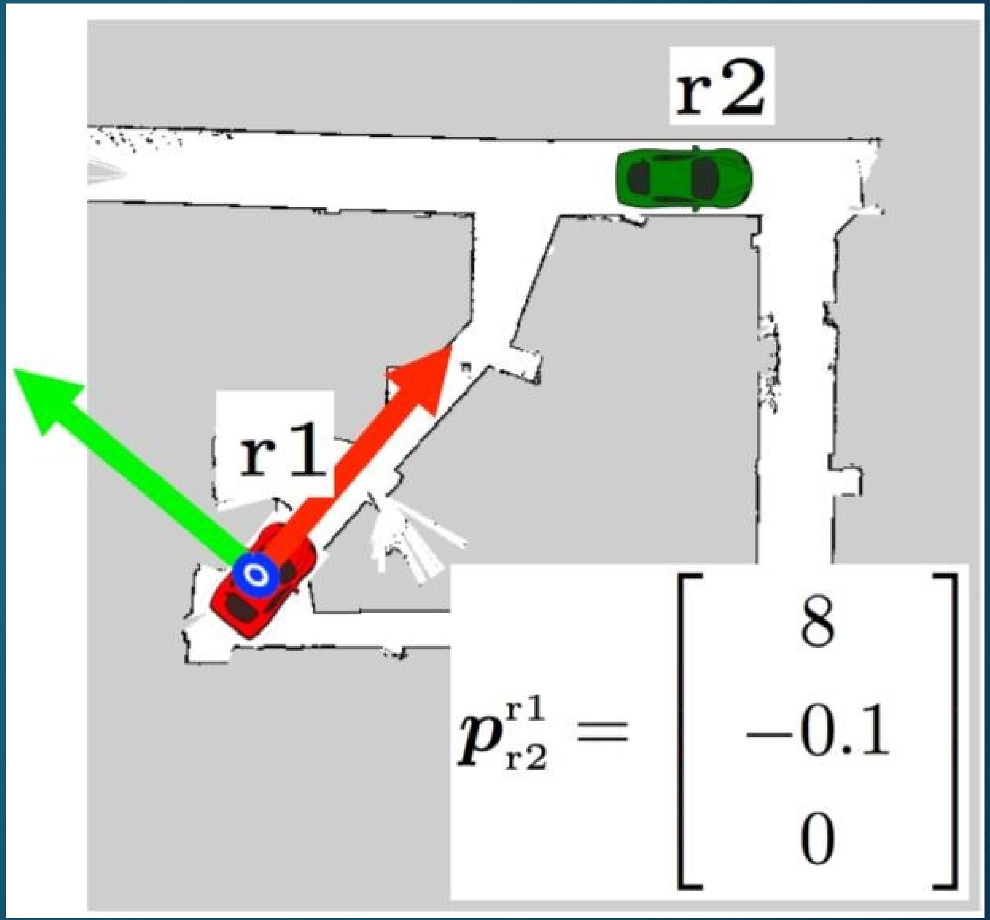


在几乎所有机器人项目中，工作的第一步通常是给目标物体赋予一个坐标系

为何需要坐标系

- 可对空间任意点的位姿采用坐标进行简洁数学表达





位姿

||

位置

+

方位角

- 而如果没有坐标系，则坐标没有存在意义

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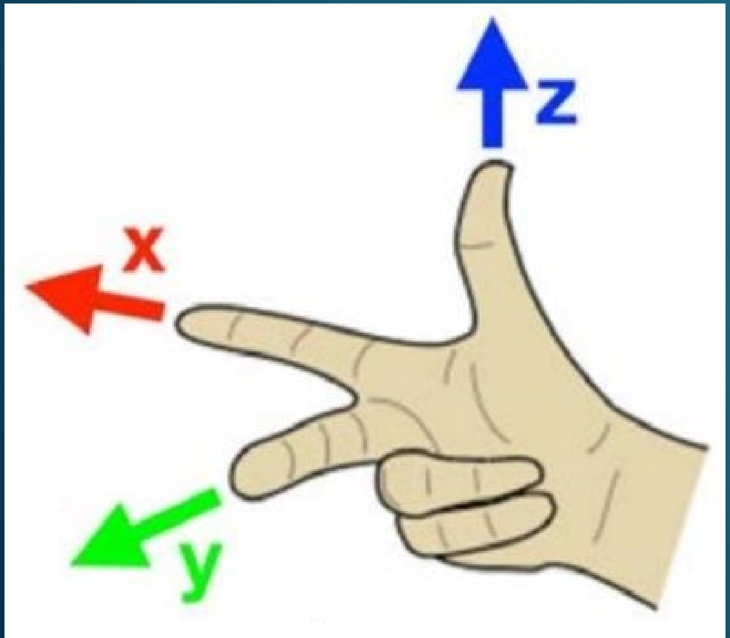
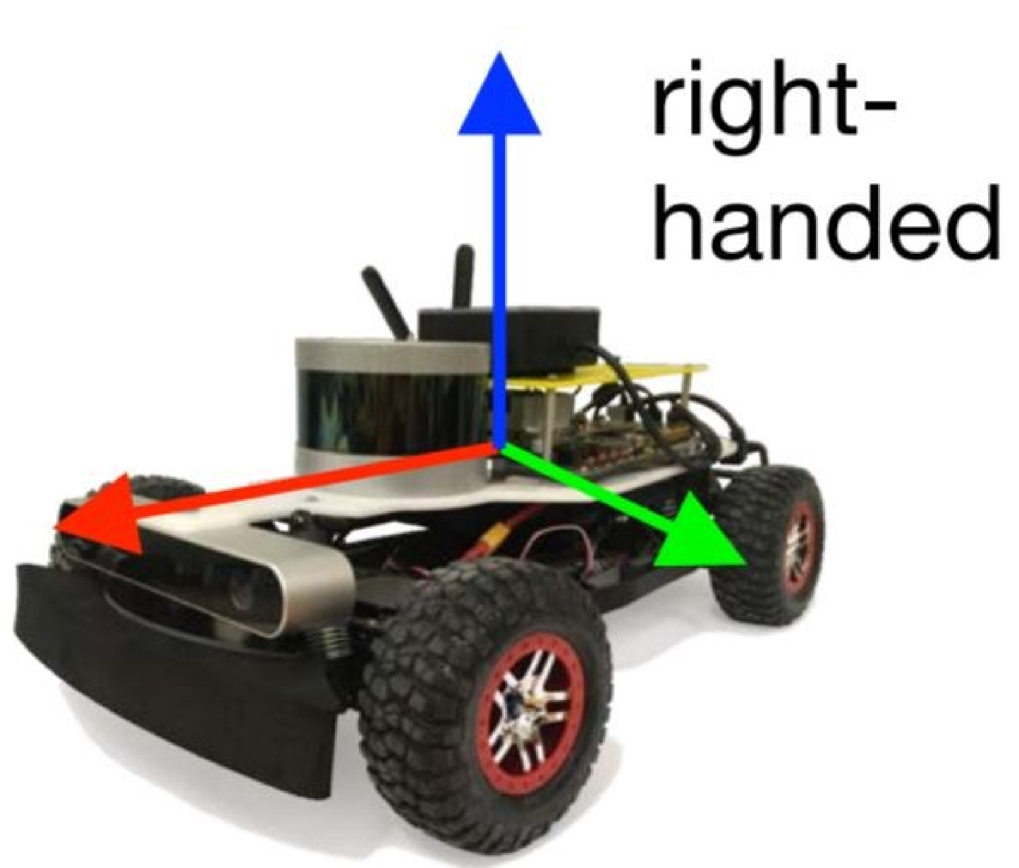
确定坐标系的右手法则

◆坐标轴的方向很重要，可由右手法则确定

* 右手的食指指向X轴正方向，中指指向Y轴正方向，
而拇指指向Z轴正方向





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坐标系与坐标变换

- 用3D摄像机拍摄到的图像是针对传感器的坐标系的外部世界表达
- 此图能否让拍摄者确定障碍物的位置？
- 此图能否让拍摄者确定自身的位置？

为了回答这些问题，必须了解不同坐标系的关系，而坐标变换则为此而生

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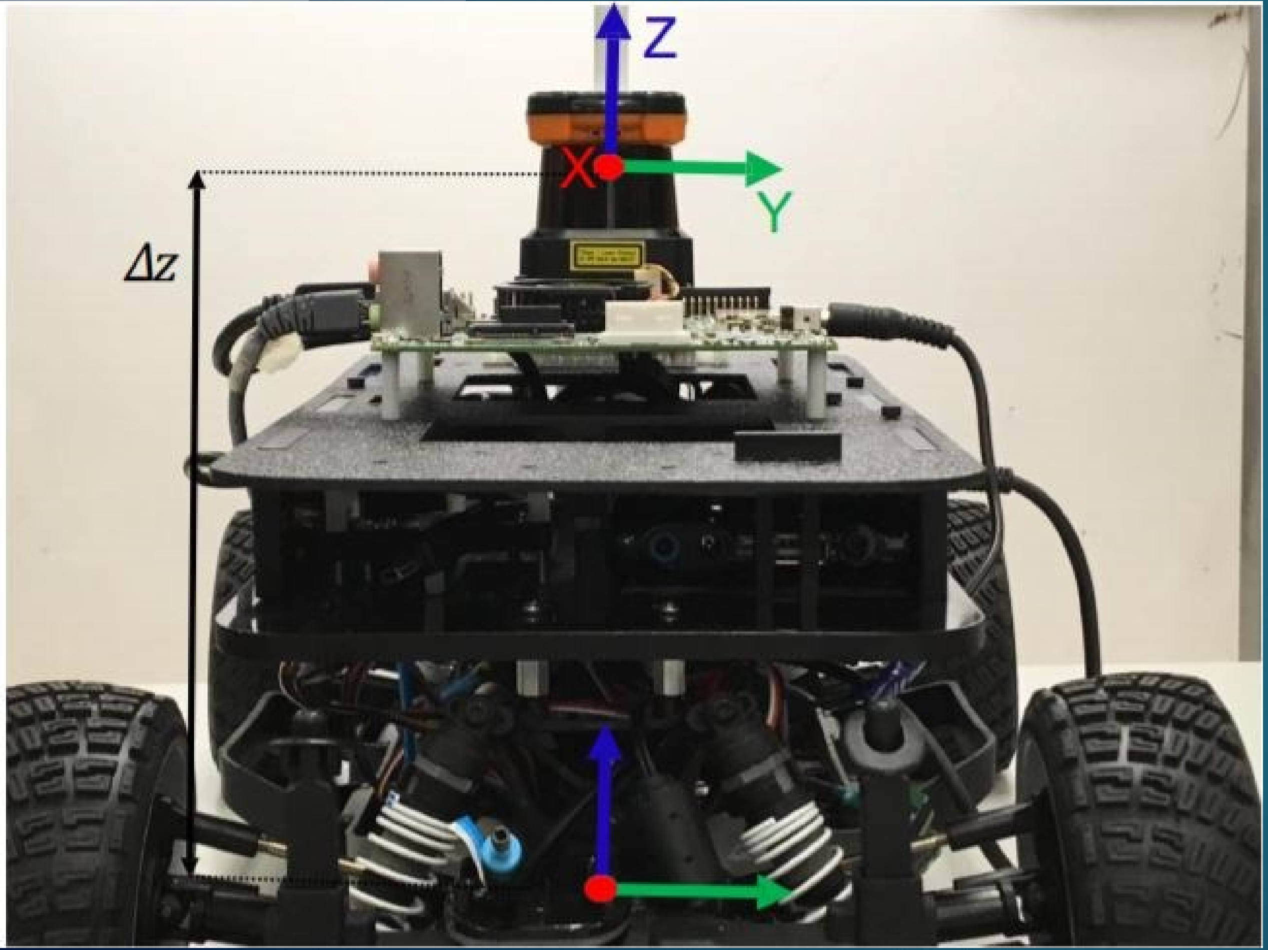
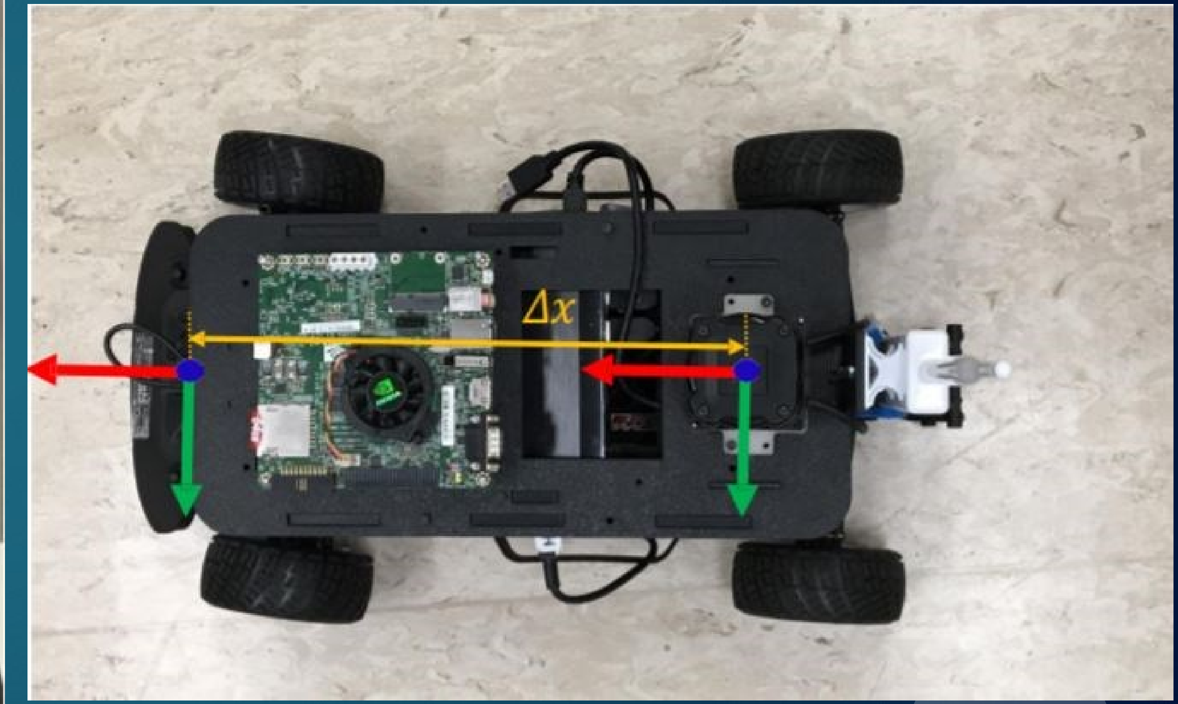
坐标系与坐标变换



很明显，来自激光雷达的数据，并不能准确得到前方障碍物的位置，必须考虑不同坐标系带来的偏移量



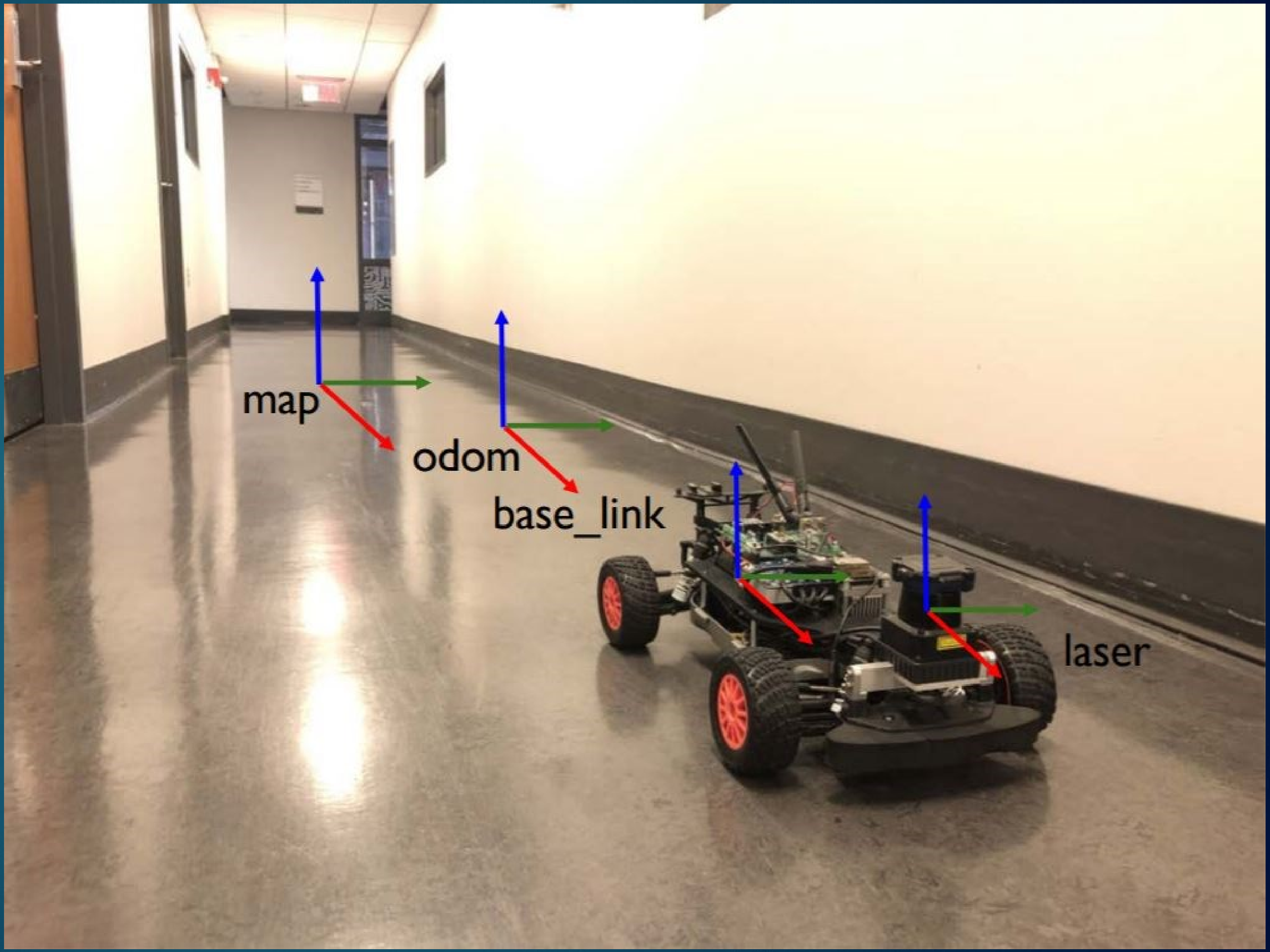
坐标系与坐标变换

- 一个典型的小型机器人通常具有如下四个坐标系：

  * map
  * base_link
  * laser
  * odom

ROS中，通常 RGB -> XYZ

Laser坐标系的意义：
　　机器人了解外部的窗口
Map坐标系的意义：
　　机器人在外部世界的位姿



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各个坐标系具体意义

◆ **Map**

* 当用户建立map坐标系时，原点设在外部环境任意位置

* 该坐标系代表机器人周边的环境

◆ **base_link**

* 通常定义在底盘上某个点，此处在底盘尾部

* 固定在机器人上不变，随机器人相对map行动

◆ **Laser**

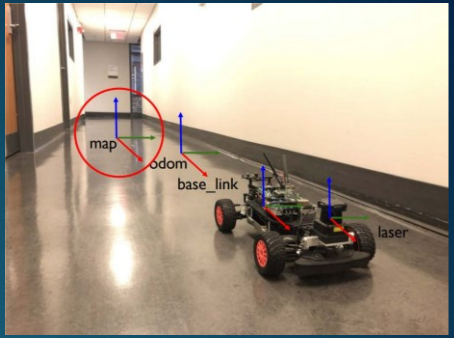
* 是激光传感器感知数据的坐标系

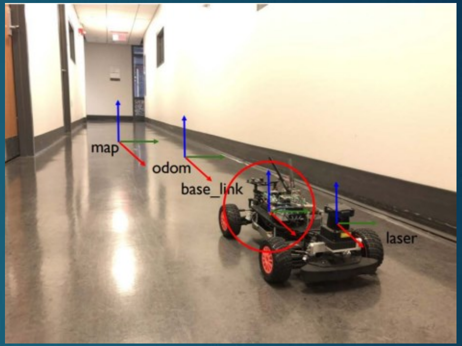
* 和base_link一样，相对机器人固定，相对map运动

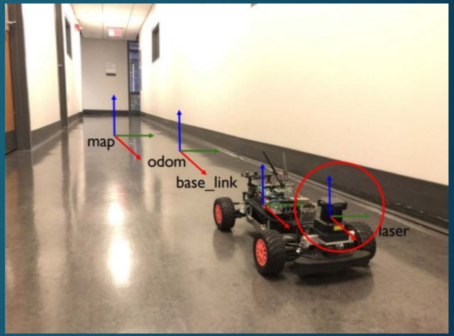
◆ **odom**

* 是里程计传感器数据的坐标系

* 相对map固定，原点是机器人开始运动的地点







坐标转换是二个坐标系之间的转换，是二者的相对关系

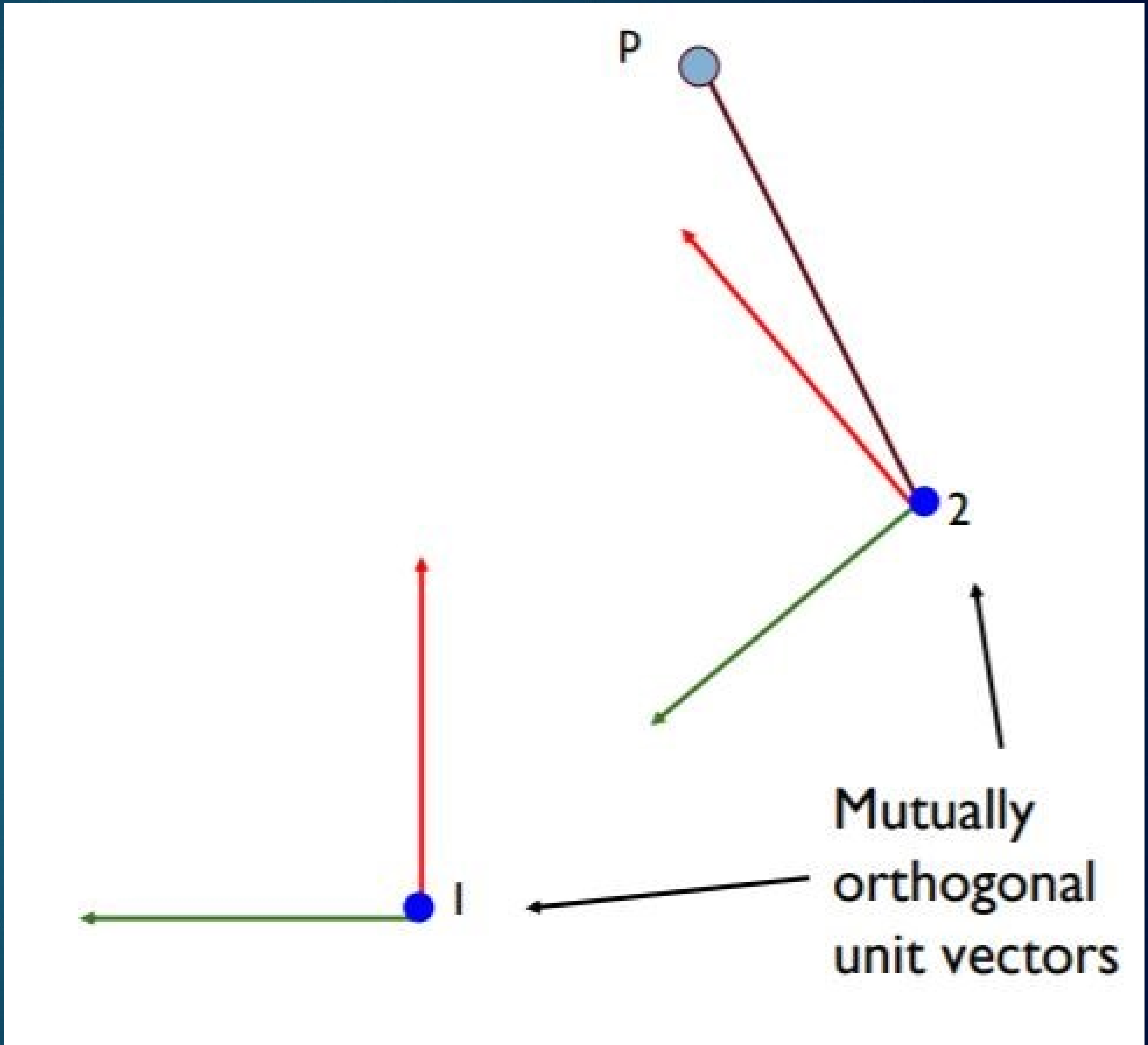
刚体坐标变换

◆ 目标：

已知P点在坐标系1中的表达，
希望它也能在坐标系2中表达

问题：如何从坐标系1得到坐标系2？

Answer: 平移 + 旋转



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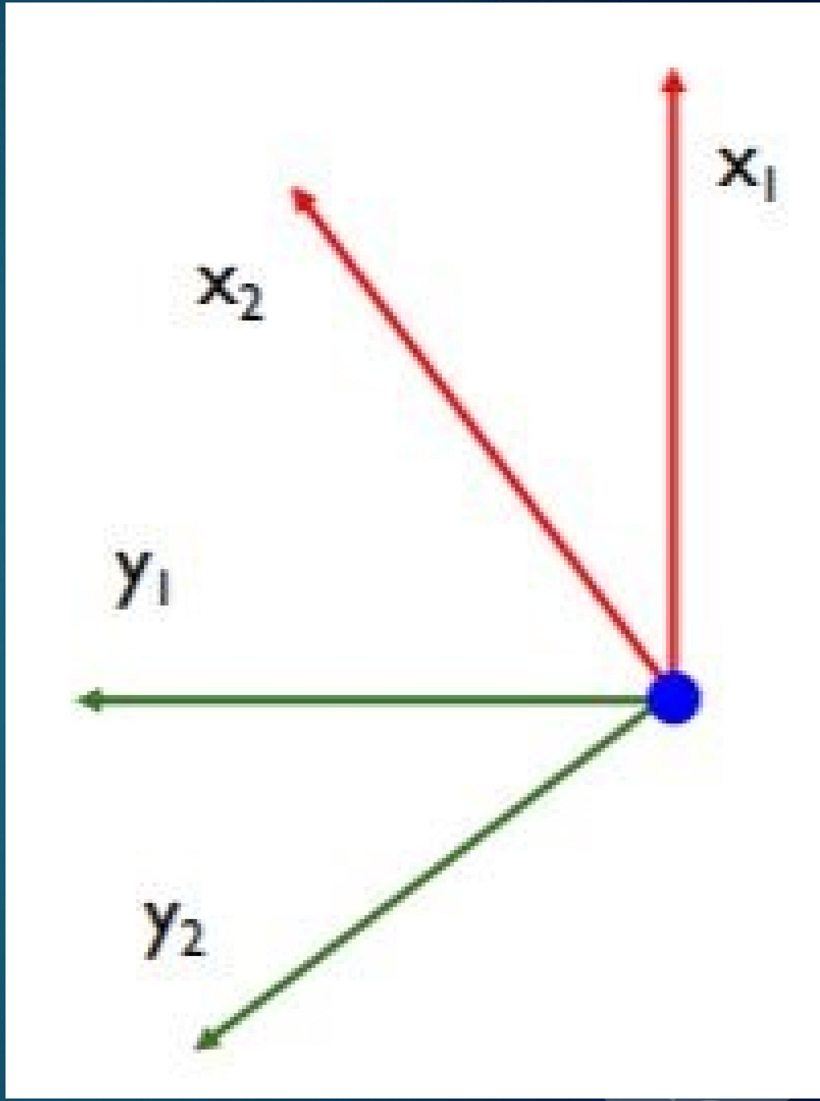
旋转矩阵

X1， Y1， X2， Y2为各自坐标轴上的单位向量

如下公式将X2，Y2向量在X1-Y1坐标系中表示了出来：

$$\mathbf{x}_2 = R_{11}\mathbf{x}_1 + R_{21}\mathbf{y}_1$$
$$\mathbf{y}_2 = R_{12}\mathbf{x}_1 + R_{22}\mathbf{y}_1$$



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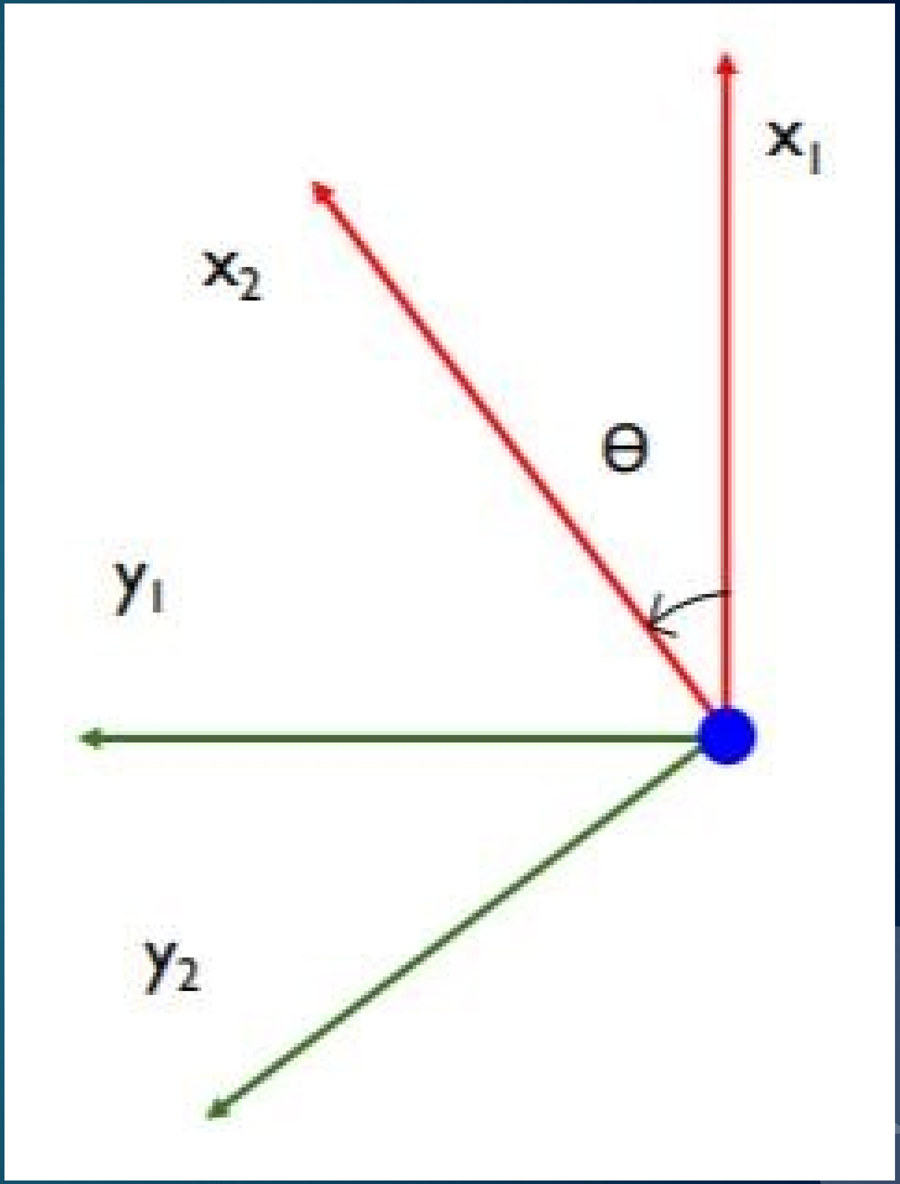
旋转矩阵

进一步整理：

$$\mathbf{x}_2 = R_{11}\mathbf{x}_1 + R_{21}\mathbf{y}_1$$
$$\mathbf{y}_2 = R_{12}\mathbf{x}_1 + R_{22}\mathbf{y}_1$$

$$\mathbf{R} = \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix}$$



旋转矩阵

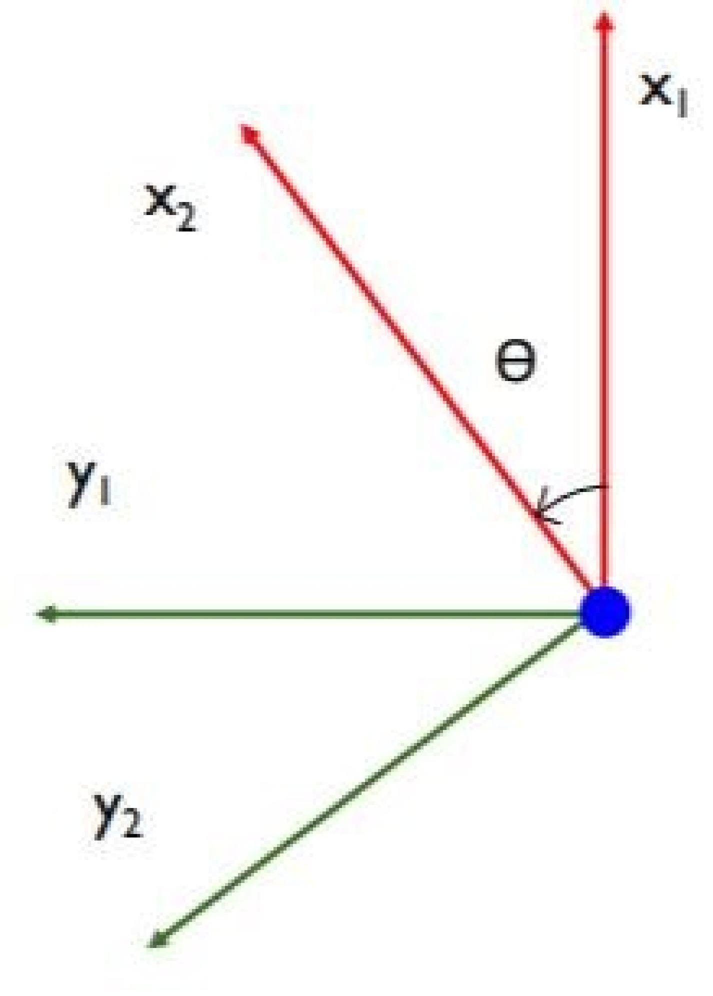
如何表达R11， R12， R21， R22 这些系数？

$$\mathbf{x}_2 = cos(\theta)\mathbf{x}_1 + sin(\theta)\mathbf{y}_1$$

$$\mathbf{y}_2 = -sin(\theta)\mathbf{x}_1 + cos(\theta)\mathbf{y}_1$$

$$\mathbf{R} = \begin{bmatrix} cos(\theta) & -sin(\theta) \\ sin(\theta) & cos(\theta) \end{bmatrix}$$

x y



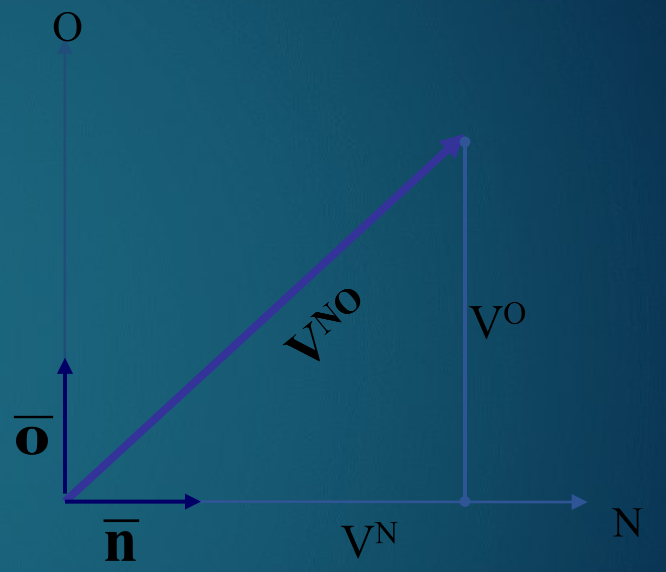
使用基本向量

基本向量是指向坐标轴正向的单位向量

$\bar{\mathbf{n}}$ Unit vector along the N-Axis

$\bar{\mathbf{o}}$ Unit vector along the O-Axis

$\left\|\mathbf{V}^{\mathbf{NO}}\right\|$ Magnitude of the V^{NO} vector



$$\bar{\mathbf{V}}^{\mathbf{NO}} = \begin{bmatrix} \mathbf{V}^{\mathbf{N}} \\ \mathbf{V}^{\mathbf{O}} \end{bmatrix} = \begin{bmatrix} \|\mathbf{V}^{\mathbf{NO}}\|\mathbf{cos\theta} \\ \|\mathbf{V}^{\mathbf{NO}}\|\mathbf{sin\theta} \end{bmatrix} = \begin{bmatrix} \|\mathbf{V}^{\mathbf{NO}}\|\mathbf{cos\theta} \\ \|\mathbf{V}^{\mathbf{NO}}\|\mathbf{cos(90-\theta)} \end{bmatrix} = \begin{bmatrix} \bar{\mathbf{V}}^{\mathbf{NO}} \bullet \bar{\mathbf{n}} \\ \bar{\mathbf{V}}^{\mathbf{NO}} \bullet \bar{\mathbf{o}} \end{bmatrix}$$

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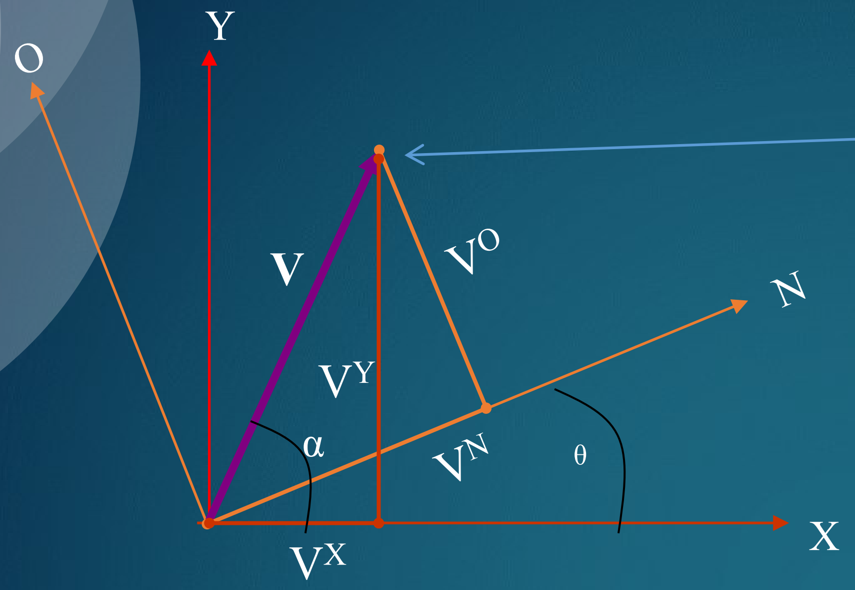

Unit vector along X-Axis

Can be considered with respect to the XY coordinates or NO coordinates

$$\|\bar{V}^{XY}\| = \|\bar{V}^{NO}\|$$

$$V^X = \|\bar{V}^{XY}\|\cos\alpha = \|\bar{V}^{NO}\|\cos\alpha = \bar{V}^{NO} \bullet \bar{x}$$

$$V^X = (V^N * \bar{n} + V^O * \bar{o}) \bullet \bar{x}$$

(Substituting for V^{NO} using the N and O components of the vector)

$$V^X = V^N(\bar{x} \bullet \bar{n}) + V^O(\bar{x} \bullet \bar{o})$$

$$= V^N(\cos\theta) + V^O(\cos(\theta + 90))$$

$$= V^N(\cos\theta) - V^O(\sin\theta)$$

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Similarly….

$$V^Y = \|\bar{V}^{NO}\| \sin\alpha = \|\bar{V}^{NO}\| \cos(90-\alpha) = \bar{V}^{NO} \bullet \bar{y}$$

$$V^Y = (V^N * \bar{n} + V^O * \bar{o}) \bullet \bar{y}$$

$$V^Y = V^N(\bar{y} \bullet \bar{n}) + V^O(\bar{y} \bullet \bar{o})$$

$$= V^N(\cos(90-\theta)) + V^O(\cos\theta)$$
$$= V^N(\sin\theta) + V^O(\cos\theta)$$

So….

$$V^X = V^N(\cos\theta) - V^O(\sin\theta)$$

$$V^Y = V^N(\sin\theta) + V^O(\cos\theta)$$

$$\bar{V}^{XY} = \begin{bmatrix} V^X \\ V^Y \end{bmatrix}$$

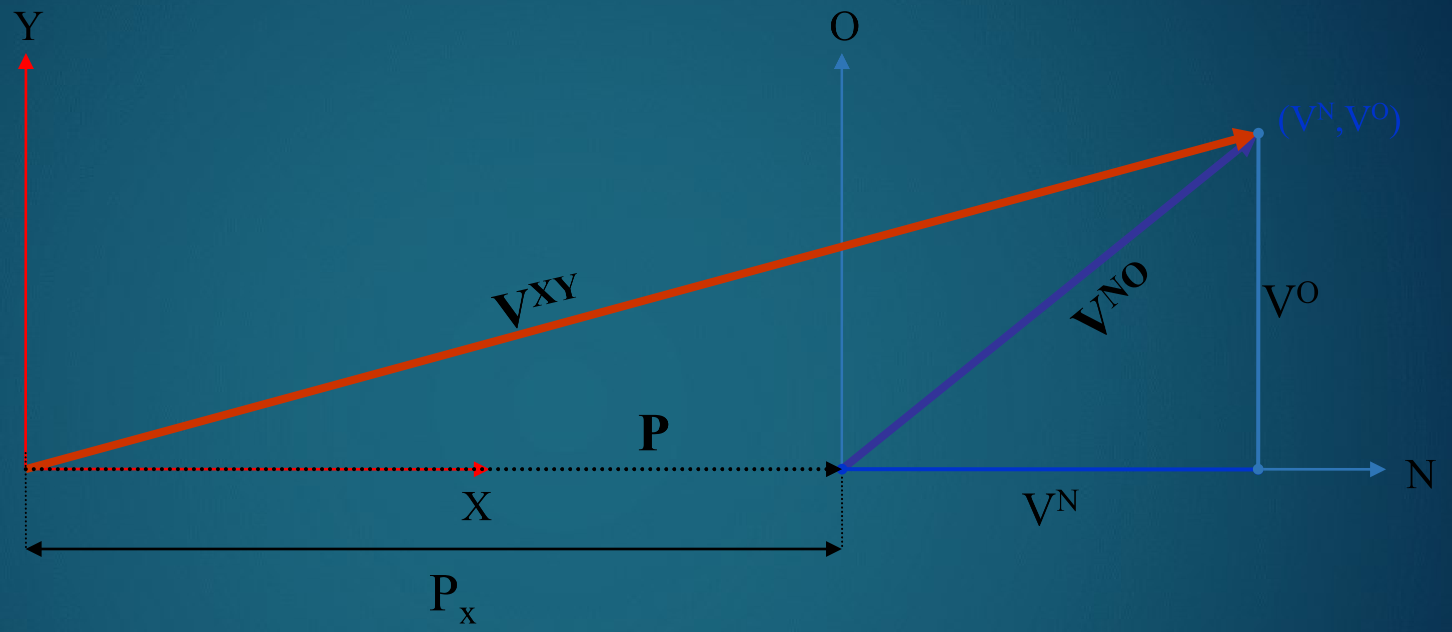
Written in Matrix Form

$$\bar{V}^{XY} = \begin{bmatrix} V^X \\ V^Y \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} V^N \\ V^O \end{bmatrix}$$

Rotation Matrix about the z-axis

平移矩阵

沿着X轴平移



P_x = distance between the XY and NO coordinate planes

$$\bar{V}^{XY} = \begin{bmatrix} V^X \\ V^Y \end{bmatrix} \qquad \bar{V}^{NO} = \begin{bmatrix} V^N \\ V^O \end{bmatrix} \qquad \bar{P} = \begin{bmatrix} P_x \\ 0 \end{bmatrix}$$

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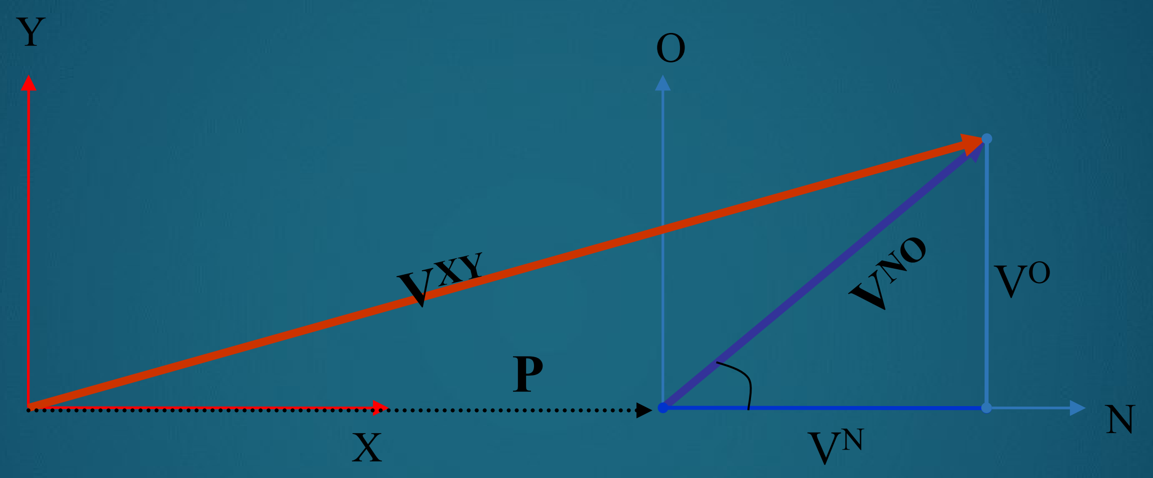
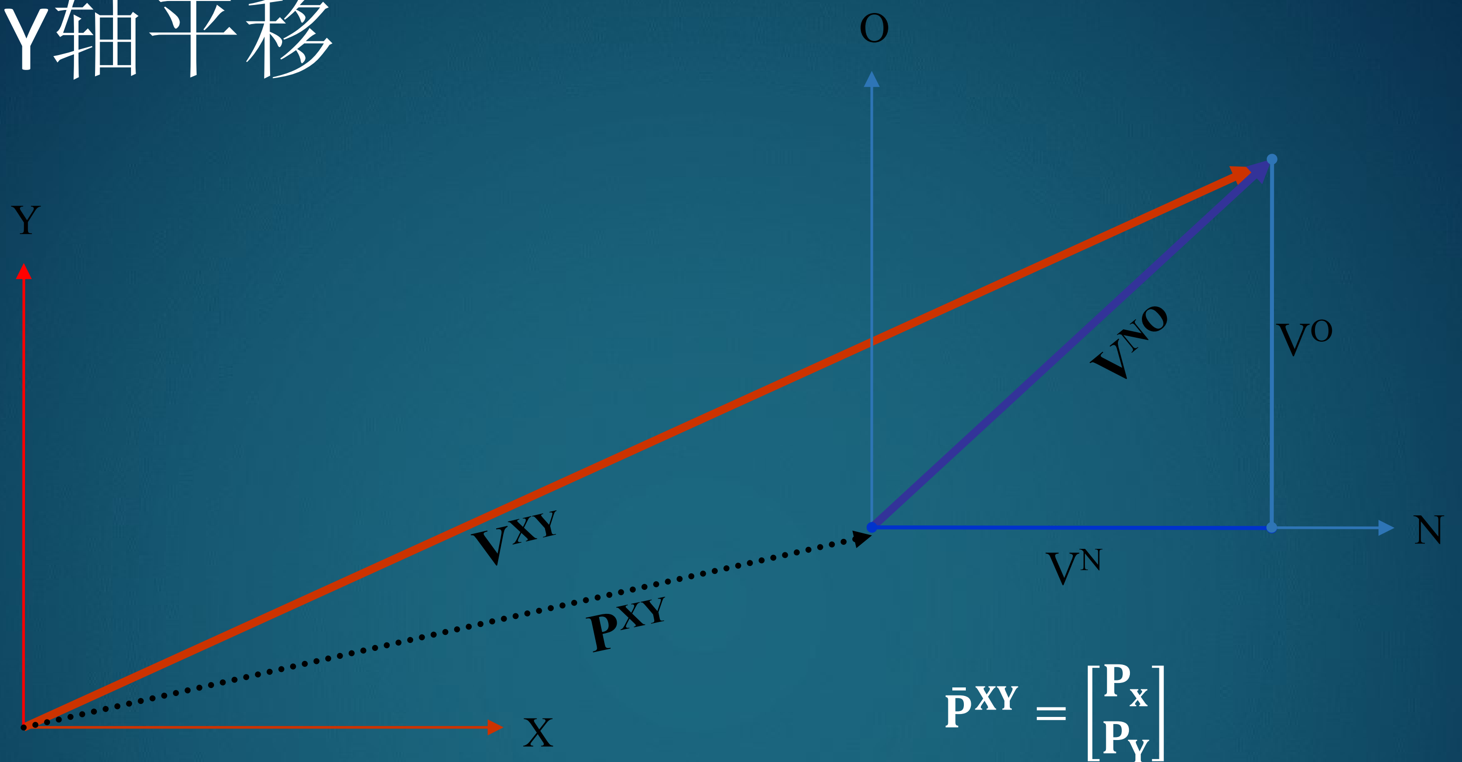
平移矩阵

使用 $\bar{V}^{NO}$ 来表示 $\bar{V}^{XY}$



$$\bar{V}^{XY} = \begin{bmatrix} P_X + V^N \\ V^O \end{bmatrix} = \bar{P} + \bar{V}^{NO}$$

同时沿X和Y轴平移



$$\bar{P}^{XY} = \begin{bmatrix} P_x \\ P_Y \end{bmatrix}$$

$$\bar{V}^{XY} = \bar{P} + \bar{V}^{NO} = \begin{bmatrix} P_X + V^N \\ P_Y + V^O \end{bmatrix}$$

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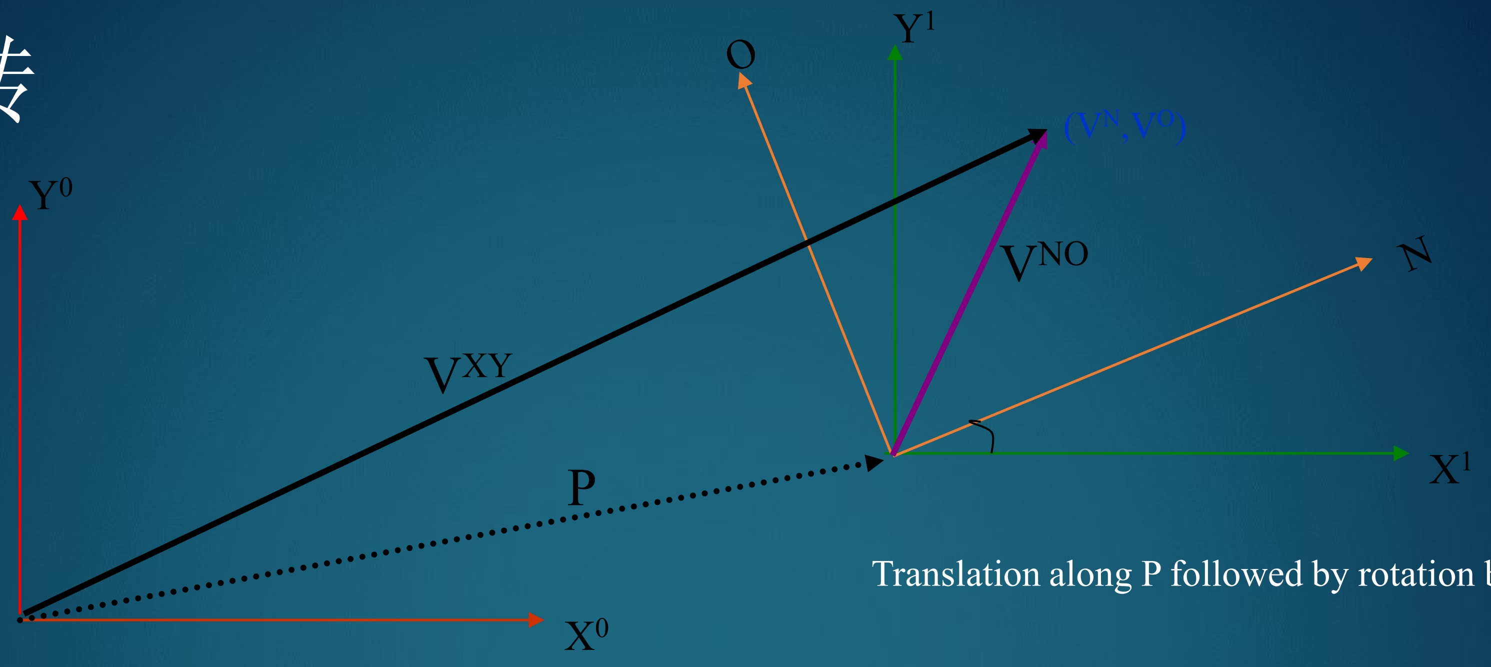
平移与旋转



Translation along P followed by rotation by θ

$$V^{XY} = \begin{bmatrix} V^X \\ V^Y \end{bmatrix} = \begin{bmatrix} P_x \\ P_y \end{bmatrix} + \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} V^N \\ V^O \end{bmatrix}$$

(Note : P_x, P_y are relative to the original coordinate frame.)

In other words, knowing the coordinates of a point (V^N,V^O) in some coordinate frame (NO) you can find the position of that point relative to your original coordinate frame (X^0Y^0).

沿Z轴平移旋转的综合表达

$$V^{XY} = \begin{bmatrix} V^X \\ V^Y \end{bmatrix} = \begin{bmatrix} P_x \\ P_y \end{bmatrix} + \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} V^N \\ V^O \end{bmatrix}$$

What we found by doing a translation and a rotation

$$= \begin{bmatrix} V^X \\ V^Y \\ 1 \end{bmatrix} = \begin{bmatrix} P_x \\ P_y \\ 1 \end{bmatrix} + \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V^N \\ V^O \\ 1 \end{bmatrix}$$

Padding with 0's and 1's

$$= \begin{bmatrix} V^X \\ V^Y \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & P_x \\ \sin\theta & \cos\theta & P_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V^N \\ V^O \\ 1 \end{bmatrix}$$

Simplifying into a matrix form

$$H = \begin{bmatrix} \cos\theta & -\sin\theta & P_x \\ \sin\theta & \cos\theta & P_y \\ 0 & 0 & 1 \end{bmatrix}$$

Homogenous Matrix for a Translation in XY plane, followed by a Rotation around the z-axis

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